# Relation between black hole entropy and quantum field spin

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Starting from the master equations of governing arbitrary spin fields, we calculate the free energy of spherically symmetric space-time due to arbitrary spin fields. Then we study the entropy of seven kinds of static space-times. The result shows that there is a linear relation between nonextreme black hole entropy and quantum field spin. Taking into account the degeneracy due to spin, we have a direct proportion relation between nonextreme black hole entropy and quantum field degeneracy. As for the extreme black hole, we find that its entropy is zero.

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To find the statistical origin of black hole entropy, 't Hooft proposed the brick-wall method in which the black hole entropy is identified with the entropy of the quantum fields surrounding the black hole itself [1]. Since the density of states approaching the horizon diverges, in order to avoid the divergence in the entropy he introduced the cutoff of the order of the Planck length, which is interpreted as the position of a "brick wall." 't Hooft himself studied the contribution to the entropy of a Schwarzschild black hole due to a scalar field. He found that the leading term of scalar field entropy is one-fourth of the area of the event horizon. After this, the method was applied to a scalar field (spin $s=0$) and a neutino field (spin $s=\frac{1}{2}$) in various black hole backgrounds [2–10], where it was shown that the leading term of neutrino field entropy is seven-eighths of the area of the event horizon. Recently, the method was extended to the electromagnetic field in a Reissner-Nordström black hole [11,12], where it was shown that the leading term of the electromagnetic field entropy is exactly twice that due to the scalar one and the electromagnetic field entropy is equal to that of the gravitational field.

In this paper, we give a calculation of the entropy of spherically symmetric black holes due to arbitrary spin fields ($s=0$ for a scalar field, $s=\frac{1}{2}$ for a neutrino field, $s=1$ for an electromagnetic field, $s=\frac{3}{2}$ for a gravitational neutrino field, $s=2$ for a gravitational field, . . . ). We found that there is a linear relation between nonextreme black hole entropy and quantum field spin. The result of the scalar field, the neutrino field, and the electromagnetic field agrees with the former. Taking into account the degeneracy due to spin, we have a direct proportion relation between nonextreme black hole entropy and quantum field degeneracy. We also found that the entropy of an extreme black hole is zero according to the brick-wall method.

In the brick-wall method, due to technical difficulties, one has to make some suitable approximation. In fact, the leading term of entropy in the brick-wall method comes from the contribution of the field near the horizon. As we all know, Hawking radiation comes from the vacuum fluctuation in the vicinity of the horizon. We think, the black hole entropy does as well. According to this idea, we improved the brick-wall method [13]. For simplicity in mathematics, we will use the improved brick-wall method in which the black hole entropy is identified with the entropy of a layer quantum field in the vicinity of the horizon.

The field equations of governing arbitrary spin fields are given by [14]

$$\begin{aligned}[][\Delta+2(s-p-1)\gamma+(2s-p)\mu]\psi_{2s-p-1}\\ -[\delta+2(s-p)\beta-(p+1)\tau]\psi_{2s-p}\\ -(2s-p-1)\nu\psi_{2s-p-2}-p\sigma\psi_{2s-p+1}=0,\end{aligned}\quad(1)$$

$$\begin{aligned}[][D+2(s-p)\varepsilon-(p+1)\rho]\psi_{2s-p}\\ -[\overline{\delta}+2(s-p-1)\alpha+(2s-p)\pi]\psi_{2s-p-1}\\ +(2s-p-1)\lambda\psi_{2s-p-2}+p\kappa\psi_{2s-p+1}=0,\end{aligned}\quad(2)$$

where $\psi_i$ is the $i$th component of the field, $s$ is the spin of the field; $i=0,1,2,\ldots,2s$; $p=0,1,2,\ldots,2s-1$; $D,\Delta,\delta,\overline{\delta}$ are differential operators of Newman-Penrose; and $\gamma,\mu,\beta$, etc., are Newman-Penrose spin coefficients. There are $2s+1$ components for a quantum field with spin $s$.

The metric of spherically symmetric space-time can be written as

$$ds^2=e^{2U(r)}dt^2-e^{-2U(r)}dr^2-R^2(r)(d\theta^2+\sin^2\theta d\varphi^2).\quad(3)$$

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$$l^{\mu}=(e^{-2U},1,0,0), \quad m^{\mu}=\frac{1}{\sqrt{2}R}\left(0,0,1,\frac{i}{\sin\theta}\right),$$

$$n^{\mu}=\frac{1}{2}(1,-e^{-2U},0,0), \quad \overline{m}^{\mu}=\frac{1}{\sqrt{2}R}\left(0,0,1,\frac{-i}{\sin\theta}\right). \tag{4}$$

Then the nonvanishing spin coefficients can be obtained,

$$\rho=-\frac{R'}{R}, \quad \mu=-\frac{R'}{2R}e^{2U},$$

$$\gamma=\frac{1}{4}(e^{2U})', \quad \alpha=-\beta=-\frac{1}{2\sqrt{2}R}ctg\,\theta, \tag{5}$$

where a prime denotes the derivative by $r$. Four differential operators are defined as

$$D=e^{-2U}\partial_t+\partial_r,$$

$$\Delta=\frac{1}{2}\partial_t-\frac{1}{2}e^{2U}\partial_r,$$

$$\delta=\frac{1}{\sqrt{2}R}\left(\partial_\theta+\frac{i}{\sin\theta}\partial_\varphi\right),$$

$$\overline{\delta}=\frac{1}{\sqrt{2}R}\left(\partial_\theta-\frac{i}{\sin\theta}\partial_\varphi\right). \tag{6}$$

Considering Eqs. (5), Eq. (1) and Eq. (2) become

$$[\Delta+2(s-p-1)\gamma+(2s-p)\mu]\psi_{2s-p-1} -[\delta+2(s-p)\beta]\psi_{2s-p}=0 \tag{7}$$

and

$$[D-(p+1)\rho]\psi_{2s-p}-[\overline{\delta}+2(s-p-1)\alpha]\psi_{2s-p-1}=0. \tag{8}$$

Setting $\delta_0=\delta R$, $\overline{\delta_0}=\overline{\delta}R$, $\alpha_0=\alpha R$, and $\beta_0=\beta R$, Eq. (7) and Eq. (8) can be rewritten as

$$R[\Delta+2(s-p-1)\gamma+(2s-p)\mu][RD-R(p+1)\rho] \times\psi_{2s-p}-[\overline{\delta_0}+2(s-p-1)\alpha_0] \times[\delta_0+2(s-p)\beta_0]\psi_{2s-p}=0 \tag{9}$$

and

$$R[D-(p+1)\rho][R\Delta+2R(s-p-1)\gamma +R(2s-p)\mu]\psi_{2s-p-1}-[\delta_0+2(s-p)\beta_0] \times[\overline{\delta_0}+2(s-p-1)\alpha_0]\psi_{2s-p-1}=0. \tag{10}$$

Set anew,

$$\psi_{2s-p}=F_{2s-p}(t,r)Y_{lm,2s-p}(\theta,\varphi),$$

$$\psi_{2s-p-1}=F_{2s-p-1}(t,r)Y_{lm,2s-p-1}(\theta,\varphi). \tag{11}$$

Then the radial equation and angular equation are obtained as

$$R[D-(p+1)\rho][R\Delta+2R(s-p-1)\gamma +R(2s-p)\mu]F_{2s-p-1}=-\frac{\lambda^2}{2}F_{2s-p-1}, \tag{12}$$

$$R[\Delta+2(s-p-1)\gamma+(2s-p)\mu][RD-R(p+1)\rho]F_{2s-p} =-\frac{\lambda^2+2}{2}F_{2s-p}, \tag{13}$$

$$\frac{\partial^2 Y_{lm,2s-p-1}}{\partial\theta^2}+\frac{1}{\sin^2\theta}\frac{\partial^2 Y_{lm,2s-p-1}}{\partial\varphi^2}+ctg\,\theta\frac{\partial Y_{lm,2s-p-1}}{\partial\theta} -\frac{2i(s-p-1)\cos\theta}{\sin^2\theta}\frac{\partial Y_{lm,2s-p-1}}{\partial\varphi} -(s-p-1)^2ctg^2\theta Y_{lm,2s-p-1}+(s-p-1)Y_{lm,2s-p-1} +\lambda^2 Y_{lm,2s-p-1}=0, \tag{14}$$

$$\frac{\partial^2 Y_{lm,2s-p}}{\partial\theta^2}+\frac{1}{\sin^2\theta}\frac{\partial^2 Y_{lm,2s-p}}{\partial\varphi^2}+ctg\,\theta\frac{\partial Y_{lm,2s-p}}{\partial\theta} -\frac{2i(s-p)\cos\theta}{\sin^2\theta}\frac{\partial Y_{lm,2s-p}}{\partial\varphi}-(s-p)^2ctg^2\theta Y_{lm,2s-p} -(s-p)Y_{lm,2s-p}+(\lambda^2+2)Y_{lm,2s-p}=0, \tag{15}$$

where $\lambda^2=[l+(s-p-1)][l-(s-p-1)+1]$ is the separation constant. $Y_{lm,2s-p}$ and $Y_{lm,2s-p-1}$ are spin-weighted spherical harmonics [15,16]. $l$ and $m$ are integers satisfying the inequalities

$$l\geqslant|s-p-1|, \quad -l\leqslant m\leqslant l. \tag{16}$$

Using the WKB approximation, i.e., letting

$$F_{2s-p-1}=e^{-i\omega t}e^{if_1(r)}, \quad F_{2s-p}=e^{-i\omega t}e^{if_2(r)}, \tag{17}$$

we get two wave numbers, respectively, from Eq. (12) and Eq. (13),

$$K_1\equiv\left|\frac{df_1}{dr}\right|=e^{-2U}\sqrt{\omega^2-e^{2U}\left[\frac{\lambda^2}{R^2}-A(r)\right]}, \tag{18}$$

$$K_2\equiv\left|\frac{df_2}{dr}\right|=e^{-2U}\sqrt{\omega^2-e^{2U}\left[\frac{\lambda^2}{R^2}-B(r)\right]}, \tag{19}$$

where

$$A(r)=2(p+1)\rho[2(s-p-1)\gamma+(2s-p)\mu],$$

$$-\frac{2}{R}\frac{\partial}{\partial r}[2R(s-p-1)\gamma+R(2s-p)\mu], \tag{20}$$

$$B(r)=-\frac{2}{R^2}-\frac{1}{R}e^{2U}\frac{\partial}{\partial r}[R\rho(p+1)],$$

$$+2(p+1)\rho[2(s-p-1)\gamma+(2s-p)\mu]. \tag{21}$$

According to semiclassical quantum theory and the improved brick wall model, the constraint imposed on wave number $K$ reads

$$n\pi=\int_{r_H+h}^{r_H+2h}K dr, \tag{22}$$

where $r_H$ is the event horizon, and $h$ is a small positive quantity, i.e., $h\ll r_H$.

The free-energy contribution of the component $\psi_{2s-p-1}$ can be obtained as

$$\begin{aligned}-\beta F_1&=\pm\sum_{mln} ln(1\pm e^{-\beta\omega})\\ &=\pm\int dl(2l+1)\int dn\ln(1\pm e^{-\beta\omega})\\ &=\int dl(2l+1)\beta\int d\omega\frac{n}{e^{\beta\omega}\pm1}\\ &=\frac{\beta}{\pi}\int_{|s-p-1|}^{l_{\max}}dl(2l+1)\int_{r_H+h}^{r_H+2h}dre^{-2U}\\ &\quad\times\sqrt{\omega^2-e^{2U}\left(\frac{\lambda^2}{R^2}-A\right)}\int_0^\infty d\omega\frac{1}{e^{\beta\omega}\pm1}\\ &=\frac{2\beta}{3\pi}\int_{r_H+h}^{r_H+2h}dre^{-4U}R^2\int_0^\infty d\omega\frac{[\omega^2+e^{2U}A(r)]^{3/2}}{e^{\beta\omega}\pm1},\end{aligned} \tag{23}$$

where the plus sign and minus sign in "$\pm$" correspond to the Fermi case and the Bose case, respectively. $\beta$ is the inverse of Hawking temperature. The extreme of integration in the variable $l$ is due to the fact $K_{1,2}$ has to be positive.

Similarly, we can get the free energy of the component $\psi_{2s-p}$:

$$-\beta F_2=\frac{2\beta}{3\pi}\int_{r_H+h}^{r_H+2h}dre^{-4U}R^2\int_0^\infty d\omega\frac{[\omega^2+e^{2U}B(r)]^{3/2}}{e^{\beta\omega}\pm1}. \tag{24}$$

To find the relation between black hole entropy and quantum field spin, we would like to consider some known static black holes.

### Schwarzschild space-time

For the Schwarzschild space-time, $e^{2U}=1-(2M/r)=(1/r)(r-r_H)$, $R=r$, where $M$ is the mass of the black hole, and $r_H$ is the radius of the event horizon. We find that the corresponding free energy of $\psi_{2s-p-1}$ and $\psi_{2s-p}$ is respectively,

$$-\beta F_1=\frac{2\beta}{3\pi}r_H^4\frac{h}{\eta^2}\int_0^\infty d\omega\frac{\omega^3}{e^{\beta\omega}\pm1}, \tag{25}$$

$$-\beta F_2=\frac{2\beta}{3\pi}r_H^4\frac{h}{\eta^2}\int_0^\infty d\omega\frac{\omega^3}{e^{\beta\omega}\pm1}. \tag{26}$$

It should be noted that we used the median theorem in the integration of $r$, hence $h<\eta<2h$. We can show that the event horizon is located at $e^{2U(r_H)}=0$, i.e., $1-(2M/r_H)=0$. So we have $e^{2U(r_H+\eta)}A(r_H+\eta)\simeq0$, and $e^{2U(r_H+\eta)}B(r_H+\eta)\simeq0$. It is obviously $F_1=F_2$. Hence we conclude that for a quantum field with spin $s$, its $2s+1$ components all contribute equal free energy to the black hole. Thus we get the whole free energy

$$F=-\frac{2}{3\pi}r_H^4\frac{h}{\eta^2}\int_0^\infty d\omega\frac{\omega^3}{e^{\beta\omega}\pm1}(2s+1), \tag{27}$$

i.e., due to the boson field part,

$$F_b=-\frac{2}{45}\frac{\pi^3}{\beta^4}r_H^4\frac{h}{\eta^2}(2s+1)\quad(s=0,1,2,\dots) \tag{28}$$

and the fermion field part,

$$F_f=-\frac{7}{8}\frac{2}{45}\frac{\pi^3}{\beta^4}r_H^4\frac{h}{\eta^2}(2s+1)\quad\left(s=\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2},\dots\right). \tag{29}$$

From the relation between entropy and free energy,

$$S=\beta^2\frac{\partial F}{\partial\beta}, \tag{30}$$

we obtain the entropy of a black hole,

$$S_b=\frac{8}{45}\frac{\pi^3}{\beta^3}r_H^4\frac{h}{\eta^2}(2s+1), \tag{31}$$

$$S_f=\frac{8}{45}\frac{\pi^3}{\beta^3}r_H^4\frac{h}{\eta^2}(2s+1). \tag{32}$$

$h$ and $\eta$ are of the same order, i.e., $h\sim\eta$. Thus we have $h/\eta^2\sim1/h$. Following 't Hooft, we choose $h/\eta^2\sim1/h=90\beta$. Considering the inverse temperature $\beta=4\pi r_H$ and the area of the event horizon $A=4\pi r_H^2$, we can write Eq. (31) and Eq. (32) as

$$S_b=\tfrac{1}{4}A(2s+1)\quad(s=0,1,2,\dots), \tag{33}$$

$$S_f = \frac{7}{8}\ \frac{1}{4}A(2s+1) \quad \left(s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\right), \tag{34}$$

where $A$ is the area of the event horizon.

Taking into account the degeneracy due to spin, we think $\psi_i$ has too many components to describe the massless fields. Therefore Eqs. (33), (34) should be written as

$$S_b = \frac{1}{4}A\omega, \tag{35}$$

$$S_f = \frac{7}{8}\ \frac{1}{4}A\omega, \tag{36}$$

where the degeneracy $\omega$ is related to spin $s$. We obviously have $\omega \leqslant 2s+1$. For example, for scalar $(s=0)$ and neutrino $(s=\frac{1}{2})$ fields we have $\omega=1$, and for electromagnetic $(s=1)$ and gravitational $(s=2)$ fields we have $\omega=2$.

### Reissner-Nordström space-time

For Reissner-Nordström space-time, $e^{2U}=1-(2M/r)+(Q^2/r^2)=(1/r^2)(r-r_H)(r-r_-)$, $R=r$, where $M,Q$ are the mass and the charge of the black hole, $r_H=M+\sqrt{M^2-Q^2}$ is the event horizon, and $r_-=M-\sqrt{M^2-Q^2}$ is the inner horizon. Similar to the Schwarzschild space-time case, we get the black hole entropy due to bosons and fermions,

$$S_b = \frac{8}{45}\frac{\pi^3}{\beta^3}\frac{r_H^6}{(r_H-r_-)^2}\frac{h}{\eta^2}\omega, \tag{37}$$

$$S_f = \frac{7}{8}\frac{8}{45}\frac{\pi^3}{\beta^3}\frac{r_H^6}{(r_H-r_-)^2}\frac{h}{\eta^2}\omega. \tag{38}$$

Substituting $h/\eta^2=90\beta$, $\beta=4\pi r_H^2/(r_H-r_-)$, and $A=4\pi r_H^2$ into Eqs. (37), (38), we get

$$S_b = \frac{1}{4}A\omega, \tag{39}$$

$$S_f = \frac{7}{8}\ \frac{1}{4}A\omega, \tag{40}$$

where $A$ is the area of the event horizon. The result changes if the black hole is extreme:

$$S_{b,\mathrm{extreme}} = \lim_{r_H\to r_-},$$

$$S_b = \lim_{r_H\to r_-}\frac{8\pi^3}{45}\left(\frac{r_H-r_-}{4\pi r_H^2}\right)^3\frac{r_H^6}{(r_H-r_-)^2}\frac{h}{\eta^2}\omega = 0,$$

$$S_{f,\mathrm{extreme}} = \lim_{r_H\to r_-}S_f = 0.$$

Thus we conclude that the entropy of the extreme Reissner-Nordström black hole is zero. We should note that we cannot take the limit in Eqs. (23), (24) since $\lim_{r_H\to r_-}\beta=\infty$. Therefore, we must take the limit in Eqs. (37), (38).

### Garfinkle-Horowitz-Strominger space-time

For Garfinkle-Horowitz-Strominger space-time, $e^{2U}=1-2M/r=(1/r)(r-r_H)$, $R=\sqrt{r(r-a)}$, $M,a$ are the mass and the coupling coefficient of the black hole, respectively, and $r_H$ is the radius of the event horizon. Similar to the Schwarzschild space-time case, we get the black hole entropy due to bosons and fermions,

$$S_b = \frac{8}{45}\frac{\pi^3}{\beta^3}r_H^3(r_H-a)\frac{h}{\eta^2}\omega, \tag{41}$$

$$S_f = \frac{7}{8}\frac{8}{45}\frac{\pi^3}{\beta^3}r_H^3(r_H-a)\frac{h}{\eta^2}\omega. \tag{42}$$

Substituting $h/\eta^2=90\beta$, $\beta=4\pi r_H$, and $A=4\pi r_H(r_H-a)$ into Eqs. (41), (42), we get

$$S_b = \frac{1}{4}A\omega, \tag{43}$$

$$S_f = \frac{7}{8}\ \frac{1}{4}A\omega, \tag{44}$$

where $A$ is the area of the event horizon.

### Garfinkle-Horne dilaton space-time

For Garfinkle-Horne dilaton space-time, $e^{2U}=[1-(r_H/r)][1-(r_-/r)]^{(1-a^2)/(1+a^2)}$, $R=r[1-(r_-/r)]^{a^2/(1+a^2)}$, where $r_H$ and $r_-$ are the event horizon and the inner horizon, $a$ is a coupling coefficient, the mass and the charge of the black hole are $M=(r_H/2)+[(1-a^2)/(1+a^2)](r_-/2)$, and $Q^2=r_Hr_-/(1+a^2)$. Similar to the Schwarzschild space-time case, we get the black hole entropy due to bosons and fermions,

$$S_b = \frac{8}{45}\frac{\pi^3}{\beta^3}r_H^4\left(1-\frac{r_-}{r_H}\right)^{(4a^2-2)/(1+a^2)}\frac{h}{\eta^2}\omega, \tag{45}$$

$$S_f = \frac{7}{8}\frac{8}{45}\frac{\pi^3}{\beta^3}r_H^4\left(1-\frac{r_-}{r_H}\right)^{(4a^2-2)/(1+a^2)}\frac{h}{\eta^2}\omega. \tag{46}$$

Substituting $h/\eta^2=90\beta$, $\beta=4\pi r_H[1-(r_-/r_H)]^{(-1+a^2)/(1+a^2)}$, and $A=4\pi r_H^2[1-(r_-/r_H)]^{2a^2/(1+a^2)}$ into Eqs. (45), (46), we get

$$S_b = \frac{1}{4}A\omega, \tag{47}$$

$$S_f = \frac{7}{8}\ \frac{1}{4}A\omega, \tag{48}$$

where $A$ is the area of the event horizon. We can easily show from Eqs. (45), (46) that the entropy of the extreme Garfinkle-Horne dilaton black hole is zero.

### Gibbons-Moeda dilaton space-time

For Gibbons-Moeda dilaton space-time, $e^{2U}=[1-(r_H/r)][1-(r_-/r)][1-(D^2/r^2)]$, $R=\sqrt{r^2-D^2}$, where $r_H$ and $r_-$ are the event horizon and the inner horizon, and $D$ is the axion-dilaton charge. The mass and the electric and the magnetic charges of the black hole are related to $r_H$, $r_-$, and $D$. Similar to the Schwarzschild space-time case, we get the black hole entropy due to bosons and fermions,

$$S_b=\frac{8}{45}\frac{\pi^3}{\beta^3}r_H^4\frac{(r_H^2-D^2)^3}{(r_H-r_-)^2}\frac{h}{\eta^2}\omega, \tag{49}$$

$$S_f=\frac{7}{8}\frac{8}{45}\frac{\pi^3}{\beta^3}r_H^4\frac{(r_H^2-D^2)^3}{(r_H-r_-)^2}\frac{h}{\eta^2}\omega. \tag{50}$$

Substituting $h/\eta^2=90\beta$, $\beta=4\pi(r_H^2-D^2)/r_H-r_-$, and $A=4\pi(r_H^2-D^2)$ into Eqs. (49), (50), we get

$$S_b=\tfrac{1}{4}A\omega, \tag{51}$$

$$S_f=\tfrac{7}{8}\,\tfrac{1}{4}A\omega, \tag{52}$$

where $A$ is the area of the event horizon. We can easily show from Eqs. (49), (50) that the entropy of the extreme Gibbons-Moeda dilaton black hole is zero.

### Horowitz-Strominger space-time

For Horowitz-Strominger space-time, $e^{2U}=[1-(r_H/r)][1-(r_-/r)]^{1/(P+1)}$, $R=r[1-(r_-/r)]^{P/[2(P+1)]}$, where $r_H$ and $r_-$ are the event horizon and the inner horizon, respectively, and $P$ is a constant. The mass and the electric and the magnetic charges of the black hole are related to $r_H$, $r_-$, and $P$. Similar to the Schwarzschild space-time case, we get the black hole entropy due to bosons and fermions,

$$S_b=\frac{8}{45}\frac{\pi^3}{\beta^3}r_H^4\left(1-\frac{r_-}{r_H}\right)^{P/(P+1)}\frac{h}{\eta^2}\omega, \tag{53}$$

$$S_f=\frac{7}{8}\frac{8}{45}\frac{\pi^3}{\beta^3}r_H^4\left(1-\frac{r_-}{r_H}\right)^{P/(P+1)}\frac{h}{\eta^2}\omega. \tag{54}$$

Substituting $h/\eta^2=90\beta$, $\beta=4\pi r_H[1-(r_-/r_H)]^{-1/(P+1)}$, and $A=4\pi r_H^2[1-(r_-/r_H)]^{P/(P+1)}$ into Eq. (53) and Eq. (54), we get

$$S_b=\tfrac{1}{4}A\omega, \tag{55}$$

$$S_f=\tfrac{7}{8}\,\tfrac{1}{4}A\omega, \tag{56}$$

where $A$ is the area of the event horizon. We can easily show from Eqs. (53), (54) that the entropy of the extreme Horowitz-Strominger black hole is zero.

### de Sitter space-time

For de Sitter space-time, $e^{2U}=1-(\lambda/3)r^2=-(\lambda/3)(r+r_H)(r-r_H)$, $R=r$, where $\lambda$ is the cosmology constant, and $r_H$ is the radius of the event horizon. Similar to the Schwarzschild space-time case, we get the black hole entropy due to bosons and fermions,

$$S_b=\frac{8}{45}\frac{\pi^3}{\beta^3}\frac{9}{4\lambda^2}\frac{h}{\eta^2}\omega, \tag{57}$$

$$S_f=\frac{7}{8}\frac{8}{45}\frac{\pi^3}{\beta^3}\frac{9}{4\lambda^2}\frac{h}{\eta^2}\omega. \tag{58}$$

Substituting $h/\eta^2=90\beta$, $\beta=2\pi\sqrt{3/\lambda}$, and $A=12\pi/\lambda$ into Eq. (57) and Eq. (58), we get

$$S_b=\tfrac{1}{4}A\omega, \tag{59}$$

$$S_f=\tfrac{7}{8}\,\tfrac{1}{4}A\omega, \tag{60}$$

where $A$ is the area of the event horizon. Clearly there is a linear relation between black hole entropy and quantum field spin. Taking into account the degeneracy due to spin, we have a direct proportion relation between nonextreme black hole entropy and quantum field degeneracy. As for the extreme black hole, we found that its entropy is zero according to the brick-wall method [17].

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